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SEVERAL DISTANCE BASED INDICES FOR COMPLEMENT OF GRAPHS

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Abstract: A graph G is said to have property P if for every pair of its adjacent vertices x and y there exists a vertex z such that z is not adjacent to x and y. In this paper, we establish an explicit formula to calculate the several graph indices for the complement of any graph G having above property. As a corollary we obtain the several graph indices for the complement of certain derived graphs.

Keywords and Phrases: Topological index, distance, derived graph.

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1. Introduction and Preliminaries

For vertices $u, v \in V(G)$, the distance between u and v in G, denoted by $d_G(u, v)$, is the length of a shortest (u, v)-path in G and let $d_G(v)$ be the degree of a vertex $v \in V(G)$. The *diameter* of the graph G is $max\{d_G(u, v)|u, v \in V(G)\}$. A *topological index* of a graph is a real number related to the graph; it does not depend on labeling or pictorial representation of a graph. There exist several types of such indices, especially those based on vertex and edge distances. One of the most intensively studied topological indices is the Wiener index.

Dobrynin and Kochetova [2] and Gutman [3] independently proposed a vertex-degree-weighted version of Wiener index called *degree distance*, which is defined for a connected graph G as $DD(G) = \frac{1}{2}\sum\limits_{u,v\in V(G)}(d_G(u)+d_G(v))d_G(u,v)$. The *additively weighted Harary index*(H_A) or *reciprocal degree distance*(RDD) is defined in [1] as $RDD(G) = \frac{1}{2}\sum\limits_{u,v\in V(G)}\frac{(d_G(u)+d_G(v))}{d_G(u,v)}$.

The generalized degree distance, denoted by $H_\lambda(G)$, is defined as $H_\lambda(G) = \frac{1}{2}\sum\limits_{u,v\in V(G)}(d_G(u)+d_G(v))d_G^\lambda(u,v)$, where λ is a any real number. If $\lambda = 1$, then $H_\lambda(G) = DD(G)$ and if $\lambda = -1$, then $H_\lambda(G) = RDD(G)$. The generalized degree distance of unicyclic and bicyclic graphs are studied by Hamzeh et. al [4, 5]. Also they are given the generalized degree distance of Cartesian product, join, symmetric difference, composition and disjunction of two graphs. The first Zagreb index is defined for a connected graph G as $M_1(G) = \sum\limits_{u\in V(G)}d_G(u)^2 = \sum\limits_{uv\in E(G)}(d_G(u)+d_G(v))$. The Zagreb indices are found to have applications in QSPR and QSAR studies as well.

2. Main results

Let G be (n,m)-graph (that is, G has n vertices and m edges). The complement of G, denoted by $\overline{G}$, is a simple graph on the same set of vertices of G in which two vertices u and v are adjacent in $\overline{G}$ if and only if they are nonadjacent in G. Obviously, $E(G)\cup E(\overline{G}) = E(K_n)$ and $\overline{m} = \left|E(\overline{G})\right| = \frac{n(n-1)}{2} - m$. The degree of a vertex v in G is denoted by $d_G(v)$; the degree of the same vertex in $\overline{G}$ is given by $d_{\overline{G}}(v) = n-1-d_G(v)$. A graph G is said to have property P if for every pair of its adjacent vertices x and y there exists a vertex z such that z is not adjacent to x and y. If G has property P, then $\overline{G}$ is connected and diameter of $\overline{G}$ is 2. In this section, we obtain the results for generalized version of degree distance and product degree distance for complement of a given graph with property P.

Lemma 2.1. *Let G be a (n,m) graph. Then $\overline{M}_1(G) = 2m(n-1) - M_1(G)$ and $\overline{M}_2(G) = 2m^2 - M_2(G) - \frac{M_1(G)}{2}$.*

Theorem 2.2. *Let G be a (n,m) graph with property P. Then*
$H_\lambda(\overline{G}) = (n-1)(2^{\lambda+1}m + n(n-1) - 4m) - M_1(G)(2^\lambda + 1)$.

Proof. From the definition of H_λ, we have

$$H_\lambda(\overline{G}) = \frac{1}{2}\sum_{u,v\in V\overline{G}}(d_{\overline{G}}(u)+d_{\overline{G}}(v))d_{\overline{G}}^\lambda(u,v).$$

One can see that for a vertex v in G, $d_{\overline{G}}(v) = n-1-d_G(v)$. Further for any vertices u and v in $\overline{G}$, the distance between these vertices are 2 when u and v are adjacent in G and 1 when u and v are non-adjacent in G. Hence

$$\begin{aligned}
H_\lambda(\overline{G}) &= \sum_{uv\in E(G)} \Big((n-1)-d_G(u)+(n-1)-d_G(v)\Big)2^\lambda \\
&+ \sum_{uv\notin E(G)} \Big((n-1)-d_G(u)+(n-1)-d_G(v)\Big) \\
&= 2^\lambda \sum_{uv\in E(G)} \Big(2(n-1)-(d_G(u)+d_G(v))\Big) \\
&+ \sum_{uv\notin E(G)} \Big(2(n-1)-(d_G(u)+d_G(v))\Big) \\
&= 2^\lambda[2(n-1)m - M_1(G)] + 2(n-1)(\frac{n(n-1)}{2} - m) - \overline{M_1}(G) \\
&= 2^{\lambda+1}(n-1)m - 2^\lambda M_1(G) + n(n-1)^2 - 2(n-1)m - \overline{M_1}(G) \\
&= (n-1)[2^{\lambda+1}m + n(n-1) - 2m] - 2^\lambda M_1(G) - \overline{M_1}(G) \\
&= (n-1)(2^{\lambda+1}m + n(n-1) - 4m) - M_1(G)(2^\lambda+1).
\end{aligned}$$

By setting $\lambda = 1$ and -1, in above theorem, we have the following corollary.

Corollary 2.3. *Let G be a (n,m) graph with property P. Then $DD(\overline{G}) = n(n-1)^2 - 3M_1(G)$ and $RDD(\overline{G}) = (n-1)[n(n-1)-3m] - \frac{3}{2}M_1(G)$.*

3. Line Graph

The Line graph of G denoted by $L(G)$ is the graph whose vertices correspond to the edges of G with two vertices in L being adjacent if and only if the corresponding edges in G are incident. Observe that $|V(L(G))| = |E(G)|$ and $|E(L(G))| = \frac{1}{2}M_1(G) - |E(G)|$. See that $M_1(L(G)) = F(G) - 4M_1(G) + 2M_2(G) + 4m$.

Corollary 3.1. *Let $L(G)$ be the line graph of G and if $G \neq K_{1,n}$ for $n \geq 3$. Then $H_\lambda(\overline{L(G)}) = m(m-1)(2(2^\lambda-2)+(m-1)) - (2^\lambda+1)(F(G)-4M_1(G)+2M_2(G)+4m)$.*

Proof. To get a values of $H^\lambda(\overline{L(G)})$ we replacing n by m and m by $\frac{1}{2}M_1(G)+|E(G)|$ in Theorem 2.2.

By setting $\lambda = 1$ and -1 in Corollary 3.1, we obtain the following.

Corollary 3.2. *Let $L(G)$ be the line graph of G and if $G \neq K_{1,n}$ for $n \geq 3$. Then*
$(i) DD(\overline{L(G)}) = m\Big((m-1)-12\Big) - 3\Big(F(G) - 4M_1(G) + 2M_2(G) + 4m\Big)$.
$(ii) RDD(\overline{L(G)}) = m(m^2-2m+5) + 3|E(G)|(m-1) - \frac{3}{2}\Big(M_1(G)(m-5) + F(G) - 2M_2(G)\Big)$.

4. The Graph G^+

Let G^+ be the graph obtained from G by attaching a pendent vertex to each vertex of G. One can easily compute that $M_1(G^+) = M_1(G) + 4m + 2n$ and $M_2(G^+) = M_1(G) + M_2(G) + 3m + n$.

Corollary 4.1. *Let G be a (n, m) graph. Then* $H_\lambda(\overline{G^+}) = (2n-1)\Big(2^{\lambda+1}(m+n) + 2n(2n-1) - 4(m+n)\Big) - \Big(M_1(G) + 4m + 2n\Big)(2^\lambda + 1)$.

Proof. To get a values of $H^\lambda(\overline{G^+})$ and $H^*_\lambda(\overline{G^+})$ we replacing n by $2n$ and m by $m + n$ in Theorem 2.2.

By setting $\lambda = 1$ and -1 in Corollary 4.1, we obtain the following.

Corollary 4.2. *Let G be a (n, m) graph. Then*
(i)$DD(\overline{G^+}) = 2n((2n-1)^2 - 3) - 3(M_1(G) + 4m)$.
(ii)$RDD(\overline{G^+}) = (2n-1)\Big(2n(2n-1) - 3(m+n)\Big) - \frac{3}{2}\Big(M_1(G) + 4m + 2n\Big)$.

5. Subdivision graph

Subdivision graph of G is the graph $S(G)$ obtained from G by inserting a new vertex into each edge of G. Note that $|V(S(G))| = |V(G)| + |E(G)|$ and $|E(S(G))| = 2\,|E(G)|$. Further, $M_1(S(G)) = M_1(G) + 4m$ and $M_2(S(G)) = 2M_1(G)$.

Corollary 5.1. *Let $S(G)$ be the subdivision graph of G such that $G \neq P_n$, $n \leq 2$. Then* $H_\lambda(\overline{S(G)}) = (n+m-1)(4m(2^\lambda-2)+(n+m)(n+m-1))+(2^\lambda+1)(M_1(G)+4m)$.

Proof. To get a values of $H^\lambda(\overline{S(G)})$ we replacing n by $n + m$ and m by $2m$ in Theorem 2.2.

By setting $\lambda = 1$ and -1 in Corollary 5.1, we obtain the following.

Corollary 5.2. *Let $S(G)$ be the subdivision graph of G such that $G \neq P_n$, $n \leq 2$. Then*
(i)$DD(\overline{S(G)}) = (n+m)(n+m-1)^2 - 3(M_1(G) + 4m)$.
(ii)$RDD(\overline{S(G)}) = (n+m)((n+m-1)^2 - 6m) - 10m - \frac{3}{2}M_1(G)$.

6. Vertex-semi total graph

Vertex-semi total graph of G is the graph $T_1(G)$ obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it. One can see that $|V(T_1(G))| = |V(G)| + |E(G)|$ and $|E(T_1(G))| = 3\,|E(G)|$. Moreover, $M_1(T_1(G)) = 4M_1(G) + 4m$ and $M_2(T_1(G)) = 4M_2(G) + 4M_1(G)$.

Corollary 6.1. *Let T_1 be the vertex-semi total graph of G and if $G \neq P_n$ for $n \leq 3$, $G \neq K_n$ for all n and $G \neq K_{1,n}$ for $n \geq 3$. Then* $H_\lambda(\overline{T_1(G)}) = (n + m -$

$1)\Big(6m(2^{\lambda}-2)+(n+m)(n+m-1)\Big)-4M_1(G)+4m(2^{\lambda}+1)$.

Proof. To get a values of $H^{\lambda}(\overline{T_1(G)})$ we replacing n by $n+m$ and m by $2m$ in Theorem 2.2.

By setting $\lambda=1$ and -1 in Corollary 6.1, we obtain the following.

Corollary 6.2. *Let T_1 be the vertex-semi total graph of G and if $G\neq P_n$ for $n\leq 3$, $G\neq K_n$ for all n and $G\neq K_{1,n}$ for $n\geq 3$. Then*
$(i) DD(\overline{T_1(G)})=(n+m)(n+m-1)^2-12(M_1(G)+m)$.
$(ii) RDD(\overline{T_1(G)})=(n+m)((n+m-1)^2-9m)-6M_1(G)+3m$.

7. Edge-semi total graph

Edge-semi total graph of G is the graph $T_2(G)$ obtained from G by inserting a new vertex into each edge of G and by joining edges to those pairs of these new vertices which lie on adjacent edges of G. One can observe that $|V(T_2(G))|=|V(G)|+|E(G)|$ and $|E(T_2(G))|=|E(G)||+\frac{1}{2}M_1(G)$. Note that $M_1(T_2(G))=M_1(G)+M_1(L(G))+8|E(L(G))|+4m$ and $M_2(T(G))=4M_2(G)+2HZ(G)+M_2(L(G))+2M_1(L(G))+2M_1(G)-4m$.

Corollary 7.1. *Let T_2 be the edge-semi total graph of G and if $G\neq P_n$ for $n\leq 3$, $G\neq K_n$ for all n and $G\neq K_1,n$ for $n\geq 3$. Then $H_{\lambda}(\overline{T_2(G)})=(n+m-1)\Big((2m+M_1(G))(2^{\lambda}-2)+(n+m)(n+m-1)\Big)-(2^{\lambda}+1)\Big(M_1(G)+M_1(L(G))+8|E(G)|+4M_1(G)+4m\Big)$.*

Proof. To get a values of $H^{\lambda}(\overline{T_2(G)})$ we replacing n by $n+m$ and m by $(m+\frac{1}{2}M_1(G))$ in Theorem 2.2.

By setting $\lambda=1$ and -1 in Corollary 7.1, we obtain the following.

Corollary 7.2. *Let T_2 be the edge-semi total graph of G and if $G\neq P_n$ for $n\leq 3$, $G\neq K_n$ for all n and $G\neq K_1,n$ for $n\geq 3$. Then*
$(i) DD(\overline{T_2(G)})=(n+m)(n+m-1)^2-3(M_1(G)+M_1(L(G))+8|E(G)|+4M_1(G)+4m)$.
$(ii) RDD(\overline{T_2(G)})=(n+m-1)((n+m)(n+m-1)-3m)-M_1(G)(\frac{3}{2}(n+m)+9)-\frac{3}{2}M_1(L(G))-12|E|$.

8. Total graph

Total graph of G is the graph $T(G)$ whose vertex set is $V(G)\cup E(G)$, with two vertices of $T(G)$ being adjacent if and only if the corresponding elements of G are adjacent or incident. Note that $|V(T(G))|=|V(G)|+|E(G)|$ and $|E(T(G))|=2|E(G)|+\frac{1}{2}M_1(G)$. Moreover, $M_1(T(G))=4M_1(G)+M_1(L(G))+8|E(L(G))|+4m$ and $M_2(T_2(G))=HZ(G)+M_2(L(G))+2M_1(L(G))+2M_1(G)-4m$.

Corollary 8.1. *Let T be the total graph of G and if $G \neq P_n$ for $n \leq 3$, $G \neq K_n$ for all n and $G \neq K_1, n$ for $n \geq 3$. Then* $H_\lambda(\overline{T(G)}) = (n+m-1)\Big((2^{\lambda+1}-4)(2m+\frac{1}{2}M_1(G))\Big) - 8M_1(G) + M_1(L(G)) + 8\,|E(G)| + 4m.$

Proof. To get a values of $H^\lambda(\overline{T(G)})$ we replacing n by $n+m$ and m by $2m+\frac{1}{2}M_1(G)$ in Theorem 2.2.

By setting $\lambda = 1$ and -1 in Corollary 8.1, we obtain the following.

Corollary 8.2. *Let T be the total graph of G and if $G \neq P_n$ for $n \leq 3$, $G \neq K_n$ for all n and $G \neq K_1, n$ for $n \geq 3$. Then*
$(i) DD(\overline{T(G)}) = (n+m)(n+m-1)^2 - 24(M_1(G) + |E(G)|) - 3(M_1(L(G)) + 4m).$
$(ii) RDD(\overline{T(G)}) = (n+m)(n+m-1)^2 - M_1(G)\Big(\frac{3}{2}(n+m) + \frac{21}{3}\Big) - \frac{3}{2}M_1(L(G)) - 6m(n+m-2) - 12\,|E(G)|\,.$

3. Conclusion:

Distance in graphs play an important role in chemistry. In this article, we have presented the results for distance based indices of complement of given graphs.

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